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**Symposium:**

***Recent Trends in Mortality and Morbidity in Ireland***

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***Projecting Population Mortality for Ireland***

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***Abstract***

Mortality data for Ireland is analysed, for recent and long-run trends, and several methods of projecting mortality rates are outlined and the results compared. Interpretation of the results suggests that it is not unreasonable to forecast that males born in calendar year 2006 have a life expectancy of 91 years (females 93 years). On the same basis, males aged 65 years in calendar year 2006 can be expected to live another 20 years on average (females 23 years). The uncertainty surrounding the forecasts is outlined.

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## **Introduction**

Increasing human longevity in the more advanced nations is one of the greatest social achievements over the last one-hundred and fifty years. In Ireland, life expectancy began to increase markedly from the last years of the nineteenth century. In 1900-1902, life expectancy at birth was 49.3 years for males and 49.6 years for females. The latest official Irish Life Table, reflecting the experience in the years 2001-2003, shows life expectancies to have increased to 75.1 years for Irish males and 80.3 years for females – a rate of increase averaging 0.26 years for males and 0.30 years for females with the passage of each calendar year over the 20<sup>th</sup> century.

It is of interest to ponder how life expectancies may change over the course of the 21<sup>st</sup> century. Aside from personal planning, a good estimate of longevity would aid the State in, say, designing and financing of a pension and healthcare system to better achieve sustainability and inter-generational equity. This paper reviews the different approaches to projecting mortality rates and applies a couple, using several different sets of assumptions, to help form a judgement on the course of mortality in Ireland over the 21<sup>st</sup> century. Specifically, we attempt to answer the question: how long will a child born in Ireland in 2006 live on average?

Mortality patterns have been changing in the developed world at a remarkable pace over the recent past. Mortality improvements have tended to accelerate at many ages and most especially at the older ages. The pattern is no different in Ireland. In particular, Ireland is now experiencing an average rate of mortality improvement higher than at any recorded period in the past. The actuarial profession, the profession that prices and reserves for mortality risks and generally advises on the prudent management of life offices and pension funds, has recently ceased publishing mortality tables with forecasts of mortality improvements because of the dramatic changes of late and the consequent very significant uncertainty inherent in any single projection. It is of interest to explore the possible long term affects of current emerging patterns, even if the resultant projections must inevitably be surrounded by considerable uncertainty.

The structure of the paper is as follows. First, we overview the different methods of projecting mortality rates. Second, we analyse mortality trends in Ireland over both long and short periods of time. [Appendix I critically reviews the underlying data from which the Irish mortality experience is inferred.] Third, we project mortality rates in Ireland and estimate future life expectancies, including for children born in 2006, by different methods and on

different bases. This section outlines the new approach to mortality forecasting adopted in the forthcoming official projection (CSO (2008)) and appendix II outlines the new approach in detail. Finally we conclude that children born in 2006 can reasonably be expected to live to their early nineties for males and mid-nineties for females.

### **Methods of Projecting Mortality**

There are several different approaches to projecting mortality rates. First, one can model the aging process and apply the model to forecast future changes. However a satisfactory model or 'law of mortality' has proved elusive, despite notable attempts by, *inter alia*, Gompertz (1825, 1860), Makeham (1860), Perks (1932), and Beard (1971) (see, for instance, Olshansky & Carnes (1997) or Forfar (2004) for a review). In fact, attempts to detect a simple mathematical formula that governs mortality over the whole of life are now largely abandoned. Accordingly, this ideal approach is not practical.

The study of the aging process and how it evolves identifies two distinct mechanisms of mortality change. The first is secular change such as better nutrition, better housing, and innovations in diagnosis and treatment of life-threatening conditions. This must be modelled as a calendar year effect. The second mechanism, somewhat more speculative, is that the aging process is essentially the accumulation of damage to the body. This theory of aging predicts that the conditions an individual lives through are recorded on their body so, for instance, early-life conditions can affect late-life mortality patterns. This mechanism suggests that year of birth should also be incorporated into projections to proxy these 'cohort' effects.

Perhaps another obvious approach to mortality forecasting is a two-step method where, first, a forecast is made of those factors that are known to significantly impact mortality (such as marital status, smoking habits and wealth) and, second, the effect on mortality rates of such changed circumstances is estimated, in addition to some underlying secular improvement. However, again, this is not feasible in practice as the explanatory variables prove just as difficult to forecast as the mortality rates themselves. In any event, the link between factors and mortality differentials tends not to be robust and the classification of the population by relevant factor is generally not available. Again, this theoretical approach is not used in practice.

We must settle on a more atheoretical approach. Typically mortality projections identify historical trends or other patterns and extrapolate those trends or patterns to a greater or lesser

degree. This broad approach can be effected in several ways. First, the observed rates of change of mortality rates over some period in the past, generally broken down by sex and age, are simply extrapolated into the future. This has been the approach used to date in Irish official forecasts (CSO (2004b)). Second, one can employ a 'targeting method', by assuming a target mortality rate of improvement from some future year and interpolating between current rates of mortality improvement rates and the targeted future rate. This method is used by [UK] Government Actuary's Department (GAD) in making forecasts overall and for each separate region of Northern Ireland, Scotland, England & Wales (GAD (2006a, 2006b)). Third, one can use parametric methods by fitting a mathematical curve that reasonably describes the mortality rates in the past, identify trends in the best fitting parameters over time, and then project the parameters and hence future mortality. Fourth, one can decompose historical mortality rates by underlying cause of death and make projections separately for each cause, typically using the main grouping of the International Classification of Diseases. For a more detailed overview of approaches applied in practice see, for instance, Wong-Pupuy & Haberman (2004) and, for approaches used in official national projections around the world see Government Actuary's Department (2001, Appendix H).

The out-turn from the different variants of atheoretical projections are similar to each other. Each identifies the primary pattern in the past is the near log-linear decline of age specific mortality rates with time (i.e., the annual rate of decline at each age tends to remain broadly constant with time). Another striking pattern is that the annual rate of decline of mortality diminishes with increasing age. These two patterns are evident in Ireland, as outlined in the next section.

All the above atheoretical approaches have another thing in common when applied in practice: the forecasts are wrong. Specifically, there has been a bias to underestimate mortality improvements. Official forecasts in many countries have a tendency to presume a reduced rate of improvement to that observed in the past, while the actual outcome has been closer to a level rate of improvement. The underlying reasons for mortality improvements in the past – better diet, vaccines and antibiotics to combat infectious diseases – are seen as 'once-off' and forecasters are reluctant to predict such major lifestyle changes or innovations in the future (Shaw (2007)). In particular, those who base their forecasts on separately modelling mortality by each main cause of death tend to overestimate mortality. Projecting mortality rates by underlying cause of death separately produces an aggregate mortality improvement rate at each age that tends to decline with increasing forecast time as, quite simply, the weights attached to each underlying cause of death changes with time to emphasize more those with a slower rate of improvement.

The inherent bias of past forecasters to predict slowing rates of improvement, quite at odds with the historical record, is itself a stylized fact that requires explanation. We speculate that it could be due in part to a perception that an overestimation of future mortality rates is more prudent or conservative than an underestimation. It could be due in part to herding behaviour - where forecasters tend to anchor their forecasts close to others and those of past forecasters - which is often observed when the forecasting exercise is especially difficult.<sup>1</sup> Or perhaps it is due to a fundamental misunderstanding of the underlying process giving rise to mortality improvements. Mortality improvements are a manifestation of man's innovation in altering his environment to better suit his needs. As such, the process of mortality improvement can be seen to be akin to technological progress or economic productivity in that they share the common driving force of man's ingenuity. No-one is seriously forecasting technological progress or economic productivity to slow. Quite the opposite, in fact, as the ingenuity of our race can be expected to be greater than at any time in the past, given that there is now more people alive, better educated, better resourced, and better incentivised to contribute to progress. On a simple analysis of the dramatic increase of measurable inputs, it would seem perverse to predict a decrease in the measurable outputs of ingenuity. The remarkably stable log-linear trend of mortality decline at each age, despite the complexity of the underlying process giving rise to it, demands, as Wilmoth (1998) observed, that "in this situation, the burden of proof lies with those who predict sharp deviations from past trends" (p. 397).

The caution of forecasters in projecting mortality improvements can be illustrated for Ireland by assumptions made in population projections in papers read to this Society. True, future population numbers are considerably less sensitive to mortality than migration or fertility assumptions, but, nevertheless, it is as easy to adopt best estimate assumptions in this regard as any other. Each forecaster used a low or even zero rate of improvement and tended to be influenced by relatively very short-term trends. Geary (1935), in forecasting the population of Ireland out to the year 2016 (an 81 year forecast horizon), assumed no change from the mortality rates of Irish Life Table 1, but acknowledged that "this is an assumption which is fortunately not likely to be realised" (p. 28) and observed "there is little doubt that a figure of 70 [for life expectancy at birth] may be achieved during the next half century" (p. 29). Geary (1941) revisited that analysis in the light of the 1936 Census, updating the base mortality assumed to Irish Life Table 2, and in two of the three new projections assumed no mortality improvement. In the third, he projected mortality improvements over the next 30 years, thereafter no improvements. The mortality improvements for both sexes were only made for

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<sup>1</sup> This theory would also account for the high significance attached to official projections, despite the inherent uncertainty and therefore scope for alternative views.

ages up to 40 years for males and 66 years for females, with improvements assumed to be in line with (the higher) mortality improvements experienced by females over the decade to 1936. Knaggs & Keane (1971), in their population projections over 25 years to the year 1996, assumed that male mortality will improve marginally up to age 15 years, following the trend evident over the period 1961-66, with no improvements at higher ages. For females, mortality rates were assumed to decline in line with the age-specific rate of decline observed over the period 1961-66 up to age 80 years, with no improvements at later ages. Keating (1977), in making population projections out to 1986, assumed no improvements in mortality from the 1971 rates, on the basis that recent short-term trends (5 years or so) both in Ireland and internationally showed little change.

### **Mortality Trends in Ireland**

The continuous registration of deaths in Ireland began in 1864, with each record of death including the sex, age, cause and location of death. Regular censuses of the population in Ireland, also sub-divided by sex, age and location, dating from before 1864 allows us to construct the mortality experience in Ireland from official sources from that date. Summary statistics of life expectancies in the twenty-six counties of Ireland prior to independence are reported in the *Report of the Commission on Emigration and other Population Problems 1948-1954* (Table 79, p. 106) and formal life tables, showing how mortality varies by age and sex have been prepared from the experience 1925-7 (Irish Life Table 1) and since that time a total of fourteen have been prepared.

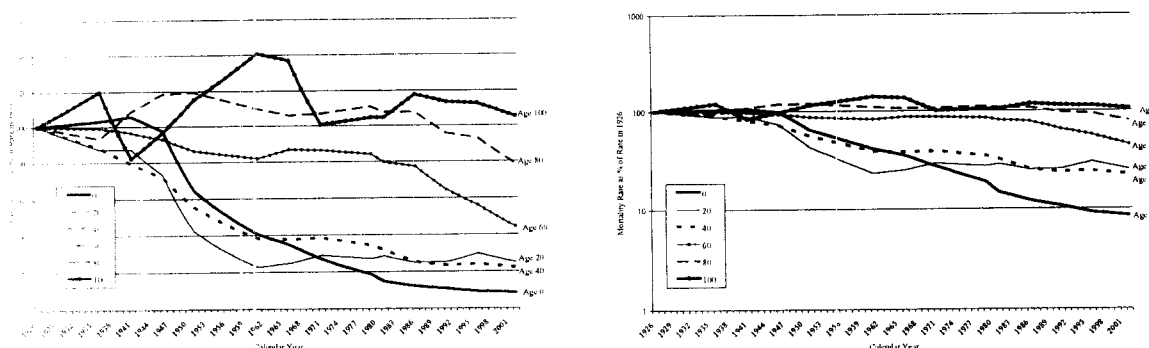
We shall restrict our analysis to mortality trends since 1926, and base it primarily on the mortality experience as it is recorded in Irish Life Tables 1-14. Appendix I considers the reliability of this source in detail. In summary, the Irish life tables give a best estimate of the mortality experienced based on the available data. Data quality has been improving over the years but there remains an issue with estimating mortality at the highest ages due to age misstatements at censuses. Ideally, mortality rates above age 85 years or so could be better estimated, in line with international best practice, using the method of extinct generations (Humphrey (1970), Thatcher (1999)). The approximate nature of mortality rates above age 85 years or so should be borne in mind in the sequel.

Trends in Irish mortality rates are now explored over the long-term, by both calendar year and year of birth, and over the short term.

*Long-term trends, by calendar year*

The long-term trends in Irish population mortality are not dissimilar to trends observed in developed economies generally. Mortality rates for either sex have declined at every age except the very advanced, with the decline being most pronounced at the early ages. *Figure 1* shows the decline in mortality from 1926 to 2002 for Irish males in the age range 0 to 100 years, separated by 20 years, on both a linear and log scale.

**Figure 1a&b: Mortality Rates of Irish Males, Selected Ages, as % of Rate in 1926.**  
**a. Percentage Scale** **b. Log-Scale**

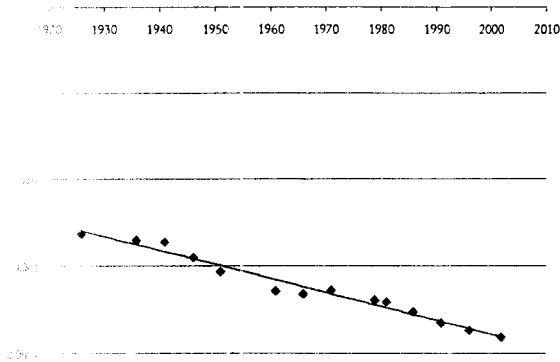


*Note:* Mortality rates at each age from ILT 1-14 Males and interpolated between census years by assuming that the age-specific mortality rates have same annualised rate of change.

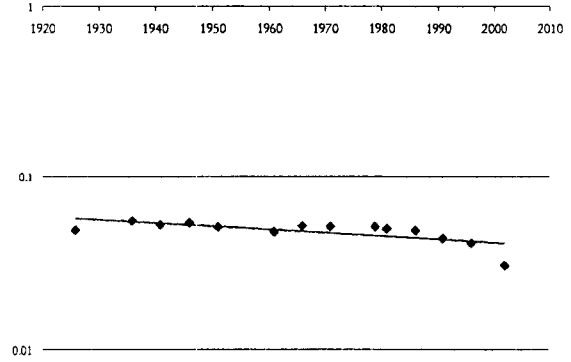
The extraordinary reductions in mortality rates at the younger ages over the 76 year period are displayed in *Figure 1*. The mortality rate at age 1 year for females showed the most improvement falling to be just one-fiftieth of its rate 1926 by 2002 (for males the 2002 rate was just 2.6% of the rate in 1926). Also evident from *Figure 1* is the pronounced age structure of mortality improvements, with the rate of improvement generally declining with increasing age so that, at very advanced ages, little or no improvement in observed.

It is not possible to see readily the timing of the improvements from *Figure 1*. *Figure 2* below plots the natural log of the mortality rate against calendar year at selected ages and shows the best fitting (log-linear) trend line. It is apparent that the trend-line captures much of the pattern in the secular improvement of mortality rates. There is nothing special about the ages selected below: a strong the log-linear pattern of mortality improvements emerges across all ages (with a better fit at the younger ages), across both sexes, over many different time periods, and across many different countries.

**Figure 2a&b: Linear Regression of Log of mortality rate ( $q_x$ ) against calendar for Irish Males at (a) age  $x=7$  years and (b) age  $x=70$  years**  
**a. Age 7 years ( $R^2$  is 0.97)**

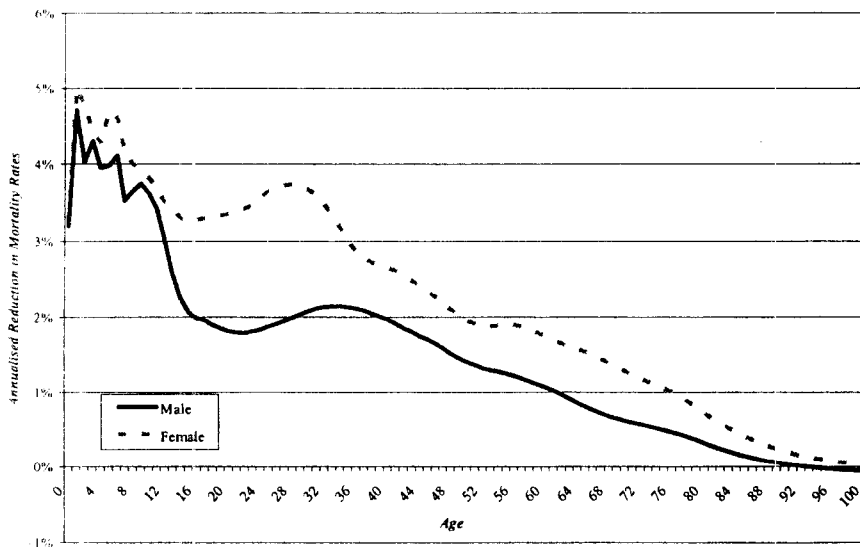


**b. Age 70 Years ( $R^2$  is 0.46)**



A more natural, and wholly equivalent way, of expressing the log-linear relationship is to say that the age-specific mortality rate tends to decline by a fixed percentage with the passing of each calendar year. *Figure 3* shows the annualised average rate of decline of mortality rates at each age for either sex over the 76 years ending 2002.

**Figure 3: Average Annualised Decline in Irish Mortality Rates, by Age and Sex, 1926-2002**



We see the declining rate of improvement with increasing age, so at ages close to 100 years, no improvement is recorded. While there are data issues associated with such high ages in Ireland (few deaths and overstatement of ages in census returns), this pattern is observed elsewhere (see Kannisto (1994)). Note that females show high rates of improvement in the late twenties and early thirties, while males show a dip in improvements in the late teens and early twenties. This is due to well-documented improvements in survival rates for females

giving birth and the lifestyle-related ‘testosterone spike’ for males and, again, both patterns are observed internationally.

Table 1 further breaks down the annualised rate of decline, this time by time period to calendar year 2002.

**Table 1: Annualised Percentage Rate of Decline in Mortality, Years Ending 2002, Various Ages, Each Sex**

Age	10 Years		20 Years		50 Years		76 Years	
	M	F	M	F	M	F	M	F
0	2.0	1.9	2.5	2.8	3.9	3.9	3.2	3.2
10	4.2	1.5	4.9	2.8	3.7	3.4	3.6	3.8
20	0.5	-0.5	0.5	0.6	0.9	3.0	1.8	3.4
30	0.5	2.8	-0.1	1.8	1.6	3.7	2.1	3.7
40	0.6	0.7	1.6	1.4	1.8	2.6	2.0	2.7
50	1.9	2.0	2.7	2.2	1.7	2.0	1.4	2.0
60	3.4	2.8	2.9	2.8	1.3	1.9	1.1	1.8
70	3.4	3.0	2.4	2.4	1.0	1.7	0.6	1.3
80	1.9	1.7	1.6	1.9	0.8	1.4	0.3	0.7
90	1.1	1.5	0.8	1.1	0.5	0.7	0.0	0.2
100	0.7	1.5	0.1	0.7	0.2	0.3	-0.1	0.0

The table highlights another pattern of mortality with time that has been observed internationally (see, for instance, Willets *et al.* (2004)). Mortality improvements, in earlier calendar years were concentrated at younger ages but, more recently, the higher improvements are at older ages. This pattern has been described as the ‘aging of mortality improvements’ (Wilmoth (1997)).

#### *Long-term trend, by year of birth*

Theories of aging also suggest that year of birth, as a proxy for the conditions the individual lived through, might also be significant in forecasting mortality. Such a ‘cohort’ effect was, in fact, noticed by the UK Government Actuary’s Department in 1995 (see Government Actuary’s Department (1995)) when they pointed out the generation born in England & Wales between 1925 and 1945 have experienced more rapid improvements than generations born earlier and later. Since that time, the cohort effect has been investigated extensively in the actuarial literature (see, for instance, Willets (1999, 2004), Willets *et al.* (2004), Richards *et al.* (2006)) and actuarial projections have modelled it explicitly (see, for instance, GAD (2006a)). A cohort effect has also been detected in Japan, centred around the year of birth 1915. The generation in England & Wales borne between 1925 and 1945 is now 63-83 years old and lower mortality rates are being observed at these later ages.

We investigate whether a cohort effect is present in Irish mortality data. To this end, we broke down the annualised rate of improvement over each decade for decennial ages and set the results out in *Table 2* below.

***Table 2: Annualised rate of improvement over each decade, 1941-2001, by decennial age, Irish Males***

Age	<u>1941</u>	<u>1951</u>	<u>1961</u>	<u>1971</u>	<u>1981</u>	<u>1991</u>	<u>2001</u>
0	-0.39	<b>4.87</b>	<b>4.47</b>	<b>4.02</b>	<b>5.93</b>	<b>3.16</b>	2.13
10	<b>2.59</b>	<b>4.91</b>	<b>5.72</b>	0.40	<b>2.59</b>	<b>6.10</b>	3.14
20	0.70	<b>6.98</b>	<b>6.01</b>	-2.44	0.20	1.20	-0.17
30	1.24	<b>5.17</b>	<b>5.87</b>	<b>1.34</b>	<b>1.22</b>	-0.51	0.14
40	1.66	3.54	<b>3.63</b>	-0.05	<b>1.91</b>	<b>3.02</b>	0.42
50	0.88	1.53	1.87	-0.31	<b>1.19</b>	<b>3.67</b>	1.81
60	0.32	1.11	0.52	-0.53	0.80	2.11	<b>3.30</b>
70	-0.10	0.28	0.66	-0.66	0.27	1.28	<b>3.02</b>
80	-1.16	-0.96	0.83	0.27	-0.07	1.16	1.68
90	0.56	-2.40	-0.78	1.23	0.18	0.46	0.95

*Note:* Mortality rates at each age from ILT 1-14 Males and interpolated between census years by assuming that the age-specific mortality rates have same annualised rate of change.

There appears to be a pattern along the diagonal of the table, consistent with a cohort effect. We notice that those born in 1931 would be 10 years of age in 1941, 20 years of age in 1951, 30 years of age in 1961, etc. The table shows that the cohorts born in 1931 and 1941 seem to have experienced a significantly lower mortality rate throughout their lives to date than preceding generations and, while generations following them also build on the decline in mortality, their rates of decrease are somewhat less spectacular. However, it is difficult to be conclusive from *Table 2*.

It is necessary to do a finer analysis. We used Irish Life tables when available otherwise we graduated mortality tables for Irish males between the ages of 12 years and 72 years, based on an average of three calendar years of deaths and population estimates in the centre year (kindly made available to us by the Central Statistics Office) using King's method and oscillatory interpolation, for each calendar year 1950 to 2001 (King (1909)). For earlier calendar years or ages outside the range, we interpolated between the closest known rates assuming a constant annual percentage rate of change in the age-specific rates. The results, for each five-year period and quinquennial age, are shown in *Table 3a*.

**Table 3a: Annualised rate of improvement over each five-year period, 1931-2001, by quinquennial ages, Irish Males**

Year	1931	1936	1941	1946	1951	1956	1961	1966	1971	1976	1981	1986	1991	1996	2001
Age															
0	-0.3	-0.3	-0.5	1.6	8.1	4.5	4.5	3.1	5.0	4.5	7.3	3.8	2.5	2.9	1.3
5	0.0	0.0	4.1	8.9	9.3	4.9	4.9	6.0	-3.2	3.0	2.3	6.7	6.9	1.9	10.3
10	2.2	2.2	3.0	2.1	7.6	5.7	5.7	1.8	-1.0	1.6	3.6	3.1	6.7	5.9	0.3
15	0.4	0.4	2.2	2.6	12.8	9.2	-0.6	-0.6	-3.5	2.3	-0.5	2.8	5.9	3.2	-8.8
20	1.4	1.4	0.0	3.4	10.5	10.1	1.6	-1.6	-3.2	0.4	0.0	2.7	-0.2	-1.4	-0.3
25	1.8	1.8	0.5	2.8	8.7	8.9	3.5	0.7	-1.0	-1.4	1.6	3.4	-2.5	-2.5	-0.1
30	1.9	1.9	0.6	3.9	6.4	8.0	3.7	4.0	-1.4	2.2	0.2	0.7	1.9	-8.0	1.7
35	1.6	1.6	1.0	3.8	5.4	6.8	2.0	3.1	4.6	-1.6	0.6	2.0	-4.2	0.9	2.7
40	1.3	1.3	2.1	2.0	5.0	5.7	1.5	0.3	-0.4	3.7	0.1	4.5	2.1	-3.1	1.4
45	0.2	0.2	2.0	1.1	4.2	5.2	1.1	1.8	-3.4	3.3	1.1	2.9	2.9	0.5	0.9
50	-0.4	-0.4	2.1	1.1	2.0	1.9	1.8	0.0	-0.6	0.8	1.6	3.6	3.0	0.8	2.6
55	0.0	0.0	1.3	0.8	1.2	2.5	-0.1	0.9	-0.4	0.5	-0.5	2.7	3.6	1.2	3.5
60	0.1	0.1	0.5	0.8	1.5	0.1	0.9	-1.2	0.1	0.5	1.1	0.6	3.9	3.2	2.9
65	-0.9	-0.9	1.2	-0.1	1.6	1.2	-0.9	-0.6	-0.6	1.0	1.1	0.2	3.2	1.7	4.0
70	-1.2	-1.2	0.9	-0.5	1.1	1.1	0.2	-1.6	0.2	0.3	0.2	0.5	2.0	1.4	4.4
75	0.0	0.0	-1.8	-1.3	1.1	1.0	1.0	-0.1	-0.6	-0.7	0.4	0.4	2.4	0.6	3.4
80	0.7	0.7	-3.0	-1.8	-0.3	0.9	0.9	0.7	-0.2	-0.5	0.3	0.0	2.1	1.0	2.3
85	0.5	0.5	-1.4	-1.6	-2.3	0.2	0.2	0.9	0.8	-0.1	0.2	-0.3	2.0	0.5	1.5
90	-0.2	-0.2	1.3	-1.7	-3.1	-0.8	-0.8	0.8	1.7	0.3	0.1	-0.6	1.9	0.5	-0.3
95	-1.0	-1.0	4.3	-2.3	-3.6	-1.4	-1.4	0.5	2.6	0.6	0.0	-1.0	1.3	-0.5	5.5
95	-1	-1	4.3	-2.3	-3.6	-1.4	-1.4	0.5	2.6	0.6	0	-1	1.3	-0.5	5.5

It is difficult to see a pattern in all the numbers. Table 3b is Table 3a adjusted by deleting any entries where the mortality improvement over the five-year period at that age is below 3% per annum. Now a pattern is more apparent.

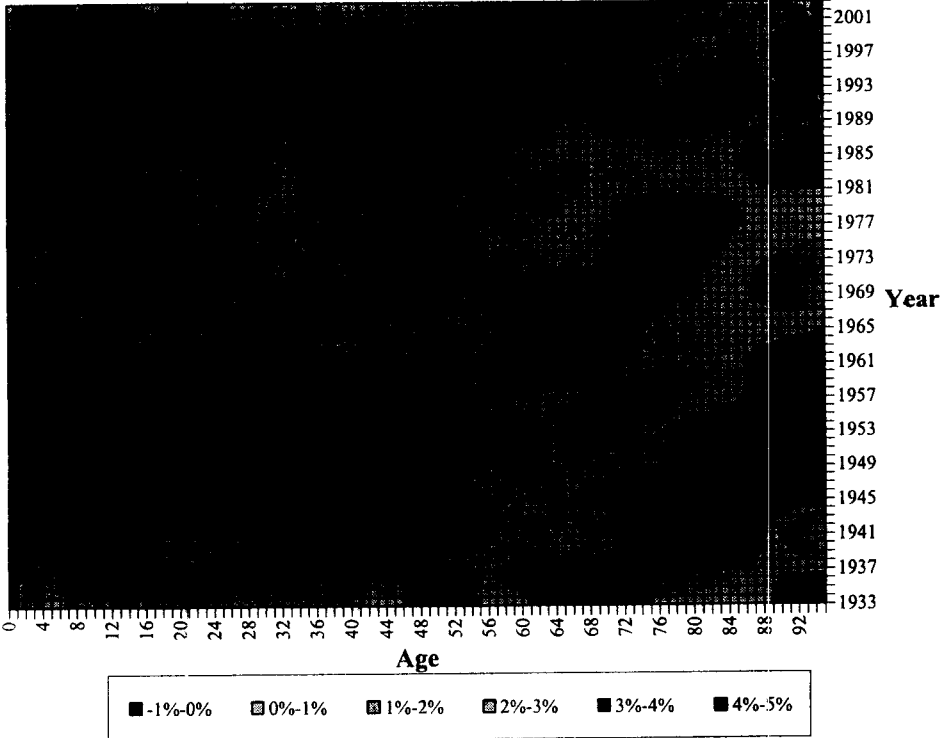
**Table 3b: Annualised rate of improvement over each five-year period which exceed 3%, 1931-2001, by quinquennial ages, Irish Males**

Year	1931	1936	1941	1946	1951	1956	1961	1966	1971	1976	1981	1986	1991	1996	2001
Age															
0					8.1	4.5	4.5	3.1	5.0	4.5	7.3	3.8			
5			4.1	8.9	9.3	4.9	4.9	6.0				6.7	6.9		10.3
10			3.0		7.6	5.7	5.7				3.6	3.1	6.7	5.9	
15					12.8	9.2							5.9	3.2	
20				3.4	10.5	10.1									
25					8.7	8.9	3.5					3.4			
30				3.9	6.4	8.0	3.7	4.0							
35				3.8	5.4	6.8		3.1	4.6						
40					5.0	5.7				3.7		4.5			
45					4.2	5.2				3.3					
50												3.6			
55													3.6		3.5
60													3.9	3.2	
65													3.2		4.0
70															4.4
75															3.4
80															
85															
90															
95			4.3												5.5

Table 3b shows the generation centred around a year-of-birth in the early 1930s are showing a step-down in mortality rates compared to previous and subsequent generations. Forecasting this trend forward would predict a fall of 3-4% per annum for those aged 65-70 years in 2001, which would follow them as they age. This rate of improvement at such advanced ages is considerably higher than that observed in the past.

One final method of visualisation of the two-dimensional data by age and year of birth is given below, the so-called 'heat-map'.

**Figure 4: Heat-map of annualized average fall in mortality rates over five calendar years, smoothed over five calendar years centred in calendar year shown. Irish Males**



*Note. Mortality rates at each age are averaged over 5 calendar years, centred in year shown, and divided by mortality rates also averaged over 5 calendar years at the same age but five calendar years earlier. Each calculated smoothed rate of improvement is a small square on the above grid and rates of the similar magnitude are then given the same colour.*

One would, of course, expect the same cohort pattern to be discernible in female mortality statistics and, as demonstrated in Tables 4a & b and the heat-map, this appears to be the case.

**Table 4a: Annualised rate of improvement over each five-year period, 1931-2001, by quinquennial ages, Irish Females**

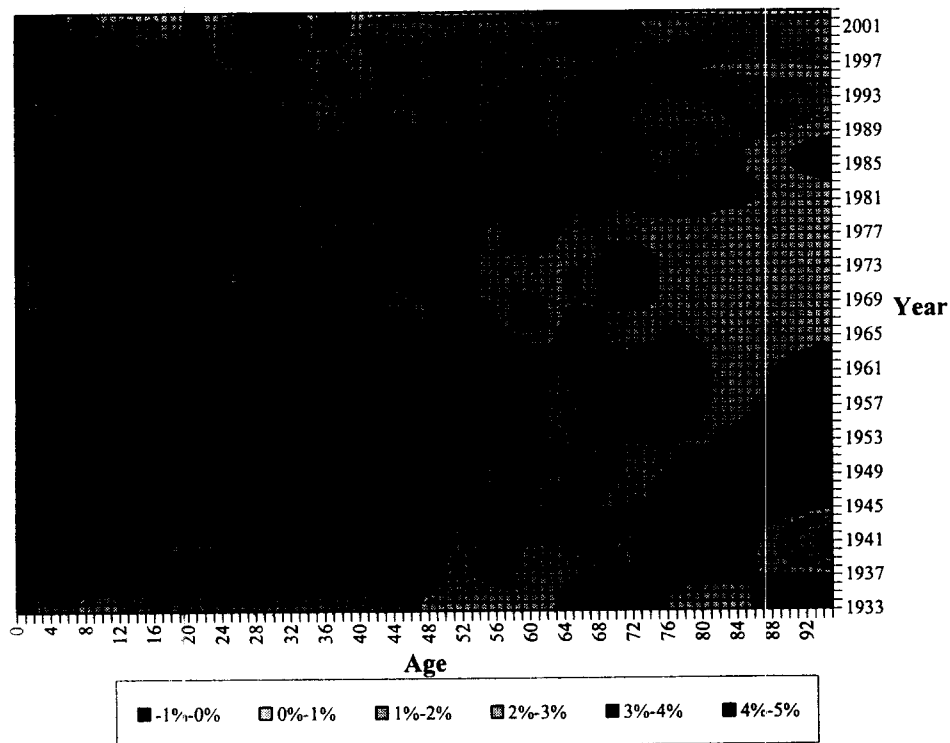
Year	1931	1936	1941	1946	1951	1956	1961	1966	1971	1976	1981	1986	1991	1996	2001
Age															
0	0.0	0.0	-0.2	0.9	8.5	4.6	4.6	3.2	4.5	4.5	6.3	4.4	3.5	1.0	2.5
5	1.4	1.4	4.8	4.8	14.1	5.1	5.1	5.9	2.9	1.7	1.8	8.9	-2.2	10.4	3.3
10	0.5	0.5	7.5	7.5	6.1	7.4	7.4	2.3	2.6	1.9	2.2	2.4	5.6	3.6	0.0
15	2.8	2.8	0.4	2.4	11.9	10.7	10.7	0.8	-5.8	5.3	2.5	4.1	-0.9	-1.7	1.4
20	1.9	1.9	-0.4	2.2	12.3	11.4	11.4	5.0	-4.6	2.9	1.8	5.8	-1.3	-1.2	0.0
25	1.6	1.6	1.6	2.2	10.3	11.3	11.3	2.7	-1.0	3.9	6.2	5.0	-6.3	0.0	2.8
30	1.3	1.3	2.8	2.2	9.3	7.9	7.9	4.4	4.0	2.9	3.3	2.6	0.0	-2.1	5.9
35	1.4	1.4	2.9	2.3	6.5	6.0	6.0	7.9	1.9	2.7	3.2	1.5	0.9	0.7	0.0
40	1.7	1.7	1.8	2.2	6.1	5.3	5.3	2.3	1.9	2.0	3.6	3.6	1.1	0.6	0.7
45	1.2	1.2	2.6	1.4	5.1	3.9	3.9	0.9	0.7	1.8	3.2	3.3	0.1	2.7	2.7
50	0.8	0.8	1.1	1.6	5.2	1.7	1.7	1.4	1.0	2.2	2.7	3.5	1.8	1.4	2.4
55	0.9	0.9	1.3	1.9	3.4	2.6	2.6	2.0	0.2	0.9	1.2	2.6	2.9	2.2	2.6
60	0.7	0.7	0.7	2.9	2.5	2.6	2.6	-0.6	0.1	1.4	1.6	1.6	3.5	2.5	3.1
65	-0.5	-0.5	1.6	1.3	3.1	2.0	2.0	2.1	-0.6	0.9	1.2	1.5	2.3	2.2	3.9
70	-0.9	-0.9	1.4	0.8	1.3	2.0	2.0	0.1	2.2	0.8	1.1	1.2	2.2	1.1	4.3
75	-0.1	-0.1	-1.3	-0.6	0.7	1.8	1.8	0.6	1.1	0.8	1.6	2.1	2.9	0.5	3.3
80	0.4	0.4	-2.5	-1.3	-0.1	1.2	1.2	0.8	0.6	0.7	1.6	1.7	2.8	0.4	2.5
85	0.1	0.1	-0.6	-1.5	-1.8	0.4	0.4	0.9	0.5	0.6	1.3	0.5	2.3	0.5	2.1
90	-0.6	-0.6	2.4	-1.7	-3.1	-0.3	-0.3	0.8	0.6	0.6	1.0	-0.6	1.9	0.6	2.0
95	-1.5	-1.5	5.9	-2.2	-4.0	-0.9	-0.9	0.7	0.8	0.5	0.9	-1.4	1.6	0.7	2.0

Source: Mortality rates at each age from ILT 1-14 Females and interpolated between census years by assuming that the age-specific mortality rates have same annualised rate of change.

**Table 4b: Annualised rate of improvement over each five-year period that exceed 3%, 1931-2001, by quinquennial ages, Irish Females**

Year	1931	1936	1941	1946	1951	1956	1961	1966	1971	1976	1981	1986	1991	1996	2001
Age															
0					8.5	4.6	4.6	3.2	4.5	4.5	6.3	4.4	3.5		
5			4.8	4.8	14.1	5.1	5.1	5.9				8.9		10.4	3.3
10			7.5	7.5	6.1	7.4	7.4						5.6	3.6	
15					11.9	10.7	10.7			5.3		4.1			
20					12.3	11.4	11.4	5.0				5.8			
25					10.3	11.3	11.3			3.9	6.2	5.0			
30					9.3	7.9	7.9	4.4	4.0		3.3				5.9
35					6.5	6.0	6.0	7.9			3.2				
40					6.1	5.3	5.3				3.6	3.6			
45					5.1	3.9	3.9				3.2	3.3			
50					5.2							3.5			
55					3.4										
60													3.5		3.1
65					3.1										3.9
70															4.3
75															3.3
80															
85															
90															
95			5.9												

**Figure 5: Heat-map of annualized average fall in mortality rates over five calendar years, smoothed over five calendar years centred in calendar year shown. Irish Females**



The cohort pattern for Irish males and females identified above is centred in a calendar year close to that identified in the UK. This is further confirmation of the general similarity of our mortality experiences (see Whelan (2006) for a comparative study, especially with Northern Ireland over much of the twentieth century).

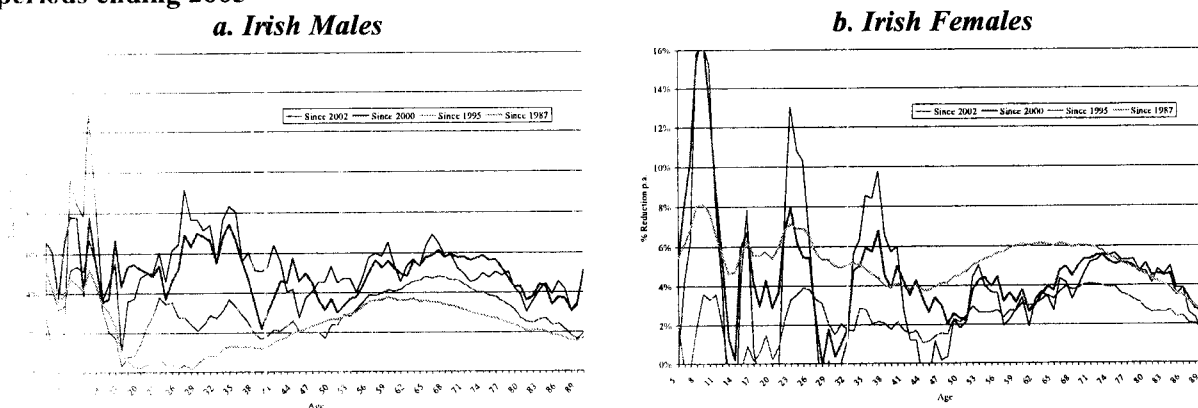
*Short-term trends in Irish mortality rates*

The long-run trends in mortality rates explored earlier treated periods up to 2002 (Irish Life Table 14). That long-run analysis suggests that we can expect (i) a continued log-linear pattern of decline of age specific rates, (ii) a continued pattern of ‘the aging of mortality improvements’, and, in particular, the cohort effect will work itself through the older ages.

Figure 6 sets out smoothed mortality rates by age observed over the three, five, ten and seventeen years ending 2005, for both males and females. First, we note that observed rates of improvements are particularly high over each of the periods, and high across all ages. Annualised rates of improvement across ages and both sexes appear to be averaging 4% or so, which is a considerably higher average rate than observed previously (see Table 1). Second,

Irish males show a markedly accelerating rate of improvement in recent years over all ages, with the smoothed average improvements over shorter periods higher than over longer periods. For females, no such accelerating pattern of improvement is evident across all ages.

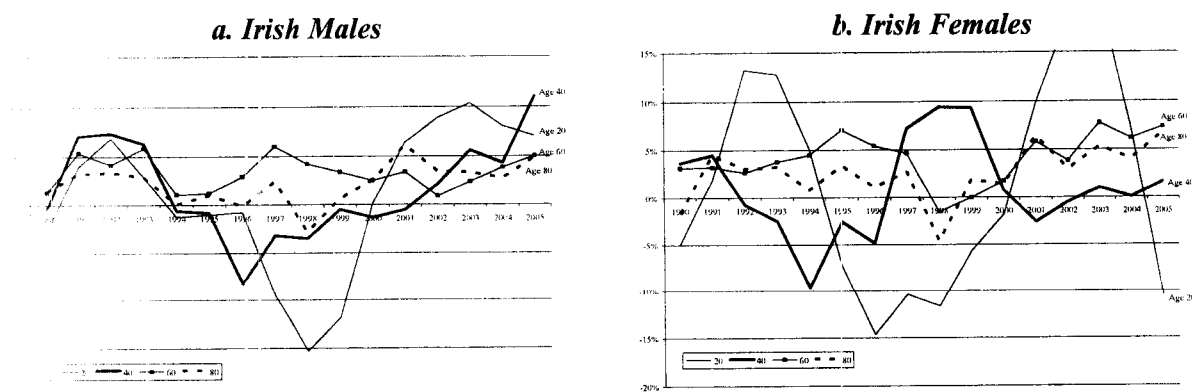
**Figure 6a & b: Smoothed Annualised Average Rate of Improvement, at each age, over periods ending 2005**



*Note: Author's computations based on deaths and estimates of population by age kindly provided by the CSO. Mortality rates in each year calculated by census method with deaths averaged over three years and then averaged over 5 years of age centred at age shown.*

Figure 7 below takes a closer look at the trend in mortality improvements with time at selected ages.

**Figure 7a & b: Annualised Average Rate of Improvement over 3 Years Ending 1990-2005, at Selected Ages**



*Note: Author's computations based on deaths and estimates of population by age kindly provided by the CSO. Mortality rates in each year calculated by census method with deaths averaged over three years.*

The time series of short-term improvements, even when smoothed, remains quite a noisy series, especially for females, so patterns are not clear-cut. From age 40 years upwards there has been an accelerating rate of improvement since 2002. Rates of improvements in recent years tend to be higher at each age than observed since 1990, except at younger female ages.

The patterns above are not completely consistent with our expectations. First, there has been a deviation from the log-linear pattern of decline by age, as rates in more recent years have declined more than the long-run average. Second, mortality rates across all ages, even the very advanced, have participated in the decline. Third, the above pattern is more marked for males than females. In particular the expected cohort effect at late ages is not apparent as calendar year improvements, giving a decline across all ages, masks it.

## **Projecting Mortality Rates**

Recent trends in mortality rates highlight the difficulties in making predictions about the future of rates with any confidence. Yet an assessment is necessary, with an appreciation of the associated uncertainty.

We treat in detail two different projection methods and analyse the forecasts from each on several different bases. The two methods are popular amongst official forecasters. In fact the Central Statistics Office applied the first method – extrapolation of the log-linear trend in the population and labour force forecasts (CSO (2004b)) and this was the method applied in all papers presented to this Society surveyed earlier. This method is known as the ‘logarithmic method’.

### *Projection Method 1: Logarithmic Method*

The logarithmic method extrapolates the log-linear trend in age-specific mortality rates observed in the past. As remarked earlier, this has been the most obvious pattern in past rates and remarkably stable over long periods of time.

Let us assume we have the mortality rate at age  $x$  in base calendar year 0. Then, by this method, the expected mortality rate at age  $x$  in calendar year  $t$  years from the base year is given by:

$$q_{x,t} = \alpha^t q_{x,0}$$

Of course  $(1 - \alpha).100\%$  gives the fixed annualised percentage decline in mortality rates expected in each future year. Taking natural logs, we get a linear relationship between the future mortality rate and time:

$$\ln q_{x,t} = t \ln \alpha + \ln q_{x,0}$$

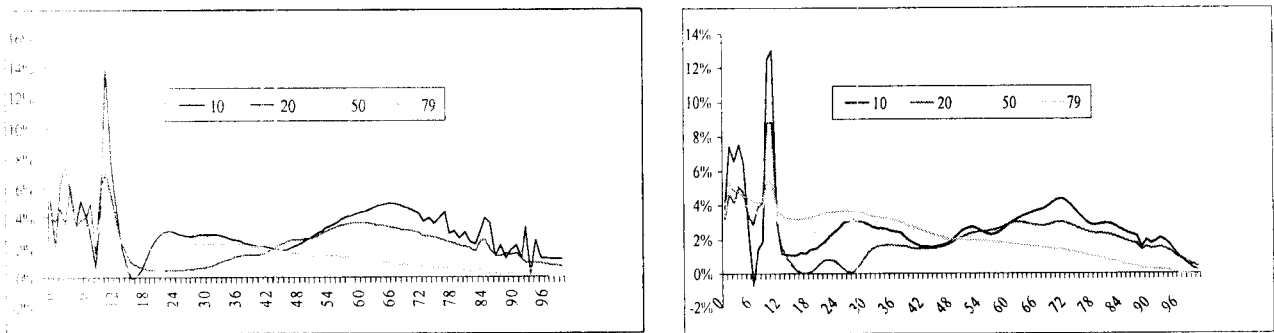
Generally the term  $\ln \alpha$  and therefore the parameter  $\alpha$  is determined by least-squares linear regression of the log of past age-specific mortality rates against calendar year (see *Figure 2* earlier). However an approximation to  $\alpha$ , generally reasonable given the strength of the log-linear trend, is to estimate the annualised rate of improvement over some suitable period in the past, i.e., over  $n$  calendar years ending in the base year by

$$\hat{\alpha} \cong \left( \frac{q_{x,-n}}{q_{x,0}} \right)^{-1/n}$$

This latter approximation has been employed by the CSO and the previous forecasts published in this Journal, so we use it here to estimate  $\alpha$ .

Materially, it is necessary to specify what period over the past is most appropriate to use to estimate  $\alpha$ . This judgement is not obvious and, as can be judged from its variability with time period (see *Figure 8*), it will significantly affect the projections. Typically, as noted earlier, forecasters used short periods of time despite then making long term forecasts. CSO (2004b) used a sixteen year period, adjusting those rates to zero that were negative.

**Figure 8a & b: Annualised Average Rate of Decline with age over 10 Years, 20 Years, 50 Years and 79 Years Ending 2005, Irish Males and Females**



*Note: Author used King's Method to graduate the crude mortality rates estimated by census method with deaths over period 2004-2006. Mortality rates in previous years from Irish Life Tables or interpolated if between Irish Life Tables.*

Both to illustrate the sensitivity of this approach to the past time period used to determine  $\alpha$  and to give some measure of the uncertainty surrounding the forecasts based on forecasting a log-linear trend, we set out in *Table 5* the projected period and cohort life expectancies at birth and at the current common retirement age of 65 years, based on extrapolating the log-linear trend observed to calendar year 2005 over, alternatively, one decade, two decades, five decades and since the first Irish Life Table in 1926.

**Table 5: Projected Period and Cohort Life Expectancies (LE), Irish Males and Females, Log-Linear Trend Extrapolation**

	Based on log-linear trend over $n$ years ending 2005, where $n=$	Period LE in 2021		Period LE in 2041		Cohort LE in 2006	
		Age 0	Age 65	Age 0	Age 65	Age 0	Age 65
Males	10 Years	82.20	20.37	87.42	23.37	93.40	19.59
	20 Years	80.85	19.29	85.08	22.50	90.78	18.68
	50 Years	78.95	17.70	81.34	19.27	84.52	17.42
	Since 1926 (79 Years)	78.48	17.10	80.21	17.90	81.63	16.86
Females	10 Years	85.51	22.83	89.65	26.18	94.94	22.62
	20 Years	84.74	22.16	88.31	25.01	93.47	21.99
	50 Years	83.92	21.26	86.58	23.16	90.09	21.05
	Since 1926 (79 Years)	83.29	20.58	85.18	21.70	87.15	20.33

*Note: Period life expectancy in 2005 at age 0 was 76.78 years for males and 81.37 years for females; at age 65 the period life expectancies were 16.43 years and 19.59 respectively.*

The table reports the projected values two types of life expectancy, period and cohort. The period life expectancy is derived from the life table constructed from the mortality experience in the indicated calendar year, i.e., it represents the expected number of years until death of an individual subject to the mortality rates of the life table constructed in that year. This is somewhat of a theoretical concept as the life table is a mixture of the mortality experience of different generations -- that of a 70 year old born 70 years ago and that of a 7 year born just 7 years ago. In order to compute cohort life expectancies one must allow for future mortality improvements. Cohort life expectancy at age  $x$  is the expected number of years that a person aged  $x$  in the given calendar year will live, and factors the expected change to mortality rates with the passage of time -- it factors in mortality improvements over the  $n$  years when the person is aged  $x + n$ . Accordingly, cohort life expectancy is the pertinent life expectancy for personal planning. In order to compute the cohort life expectancy at age 0 it is necessary to project mortality improvements over a 100 year period plus.

Let us focus on the cohort life expectancy in 2006 for a child born in that year. First, we note, for both males and females, the longer the period in the past used to determine the projected trend, the lower the life expectancy computed. This is a consequence of the accelerating trend in mortality improvements observed, which is more marked for males. Second, we note that cohort life expectancies for males differ by as much as a dozen years, depending on the past period used to determine the trend.

The principal problems in using the logarithmic method can be listed as:

- (1) The choice of past period is somewhat arbitrary but has key impact on the results. A general rule is that the longer period used, the lower the trend rate of improvements

forecast. This is because of the accelerating trend of improvement seen in recent years, especially at the older ages.

- (2) It makes no attempt to forecast the ‘cohort’ pattern of mortality improvement evident in the past.
- (3) It produces a discontinuity in (the first derivative of) mortality rates with time, as current rates are projected to make a step change in the first year of the forecast to the long term average rate.
- (4) If age-specific rates of improvement are estimated and projected for each age separately, then the method is likely to produce projected (period) mortality tables that do not progress smoothly with age and where, in fact, the monotonic increase in mortality rates at later adult ages is not always predicted. One way to overcome this problem is to smooth age-specific rates of improvement but this generally not done.

#### *Projection Method 2: Targeting Approach*

A ‘targeting approach’ has been used by, inter alia, the [UK] Government Actuary’s Department (GAD) in forecasting mortality in their national population projections and, in particular, in projecting the age and sex composition of the population of Northern Ireland to 2074 (GAD (2006b)). The targeting approach has three distinct components: (a) an estimate of short term mortality trends by age and sex, (b) a judgement of the long term rate of improvement at each age and sex from some target year in the future, and, (c) interpolating in some manner between the observed short-term trend now and the longer term trend assumed from the target year.

The GAD take the target year to be 25 years from the base year of the projections, anchor expectations about the long term rate of improvement from the target year by the rates of improvement observed over the long term past (i.e., the entire 20<sup>th</sup> century), and interpolate between the current short-term trend and the assumed long-term trend rates of improvement in a linear manner, but explicitly allowing for a cohort effect at later ages. The methodology is critically reviewed and compared with other projection methods in GAD (2001), including an evaluation of how the different methods performed in the past. Methods evaluated included the logarithmic, logit, Lee-Carter (modified and stochastic), as well as the various methodologies employed by the Continuous Mortality Investigation Bureau of the Faculty and Institute of Actuaries. The report reaches “...the clear conclusion was that there were no grounds for believing that an alternative methodology would be likely to outperform the present method” (p. 109) and that a key strength of their methodology is its ability to

incorporate the cohort effect. However, the targeting approach as employed by the GAD requires two key inputs from the user – the target date and the assumed rate of improvement in mortality rate from that target date – and the results are obviously very sensitive to these user-determined assumptions. In fact, in applying this method over recent years, the GAD has been revising upward the long-term trend assumed and even the currently assumed 1.0% per annum rate of decline across all ages and both sexes from 2029 – a rate close to the average rate of improvement over 20<sup>th</sup> century - does not get the endorsement of their expert advisory panel who consider it too low (see Appendix III, p. 79 of GAD (2006a)).

The targeting method does, though, overcome most of the drawbacks of the logarithmic method: it can readily be adapted to allow for cohort effects, its projections tie-in smoothly with current trends, and it can produce period mortality tables with the right shape in each future year. In fact the forthcoming population and labour force projections of Ireland (CSO (2008))<sup>2</sup> adopt a targeting method. The approach was similar in principle to the GAD approach but somewhat different in detail, and more importantly, in the parameter adopted for long-term rate of improvement. So, like the GAD approach, mortality rates were forecast by estimating the current rate of improvement for each sex at each age and assuming that the current rate of improvement will decline over a twenty-five year period to a long-term average improvement rate not dissimilar to that observed rates in the long-term past. It turned out with the Irish data that the current rate of decline of mortality for males averaged at 5% per annum across most ages, with surprisingly little variation. For females, the current rate of decline oscillated with age about an averaged rate of 3.5% per annum. It was judged reasonable in the estimates for the long-term future to apply the same rate of decline to male and female mortality rates and, on discussion, a long term rate of 1.5% per annum was settled upon as not unreasonable for all ages up to age 90 years after calendar year 2031. The remarkably stable level of current improvements for each sex across most ages, mean that future projections can be interpreted as either cohort projections or calendar year projections, with the rate of improvement smoothly decaying by calendar year or year of birth at the same rate. It was assumed, because of the paucity of data, that there would be no mortality improvements at ages of 100 years upwards. For each year between 2005 and 2031, the mortality declines for that year were calculated by linear interpolation. For each age between age 90 years and 100 years, the rate of mortality decline for that age was estimated by linear interpolation in each future calendar year. The approach, in effect, assumes that the secular and cohort effects begin to decay after age 90 years and by age 100 years no improvement in

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<sup>2</sup> The author is a member of the expert advisory panel and produced, on the agreed basis, the mortality forecasts.

mortality rates occur. Appendix II sets out, step-by-step, how the target method was applied to project Irish population mortality from calendar year 2005, together with a brief rationale.

Table 6 below sets out the results of applying the targeting method, and gives an indication of the sensitivity of the projected life expectancies to the assumed parameters.

**Table 6: Projected Period and Cohort Life Expectancies (LE), Irish Males and Females, Targeting Approach**

	<i>Parameter</i>	<i>Period LE in 2021</i>		<i>Period LE in 2041</i>		<i>Cohort LE in 2006</i>	
		<i>Age 0</i>	<i>Age 65</i>	<i>Age 0</i>	<i>Age 65</i>	<i>Age 0</i>	<i>Age 65</i>
<i>Males</i>	Central Projection Basis	83.1	21.1	86.5	23.7	91.0	20.6
	Initial Decline Up 1.0% p.a.	84.1	21.8	87.5	24.5	91.8	21.2
	Initial Decline Down 1.0% p.a.	82.1	20.3	85.4	22.9	90.1	19.8
	Long-term Decline Up 0.5% p.a.	83.4	21.3	87.6	24.6	93.0	20.8
	Long-term Decline Down 0.5% p.a.	82.9	20.9	85.4	22.9	88.6	20.3
<i>Females</i>	Central Projection Basis	85.5	22.9	88.2	25.1	92.5	22.7
	Initial Decline Up 1.0% p.a.	86.3	23.4	89.1	25.6	93.1	23.2
	Initial Decline Down 1.0% p.a.	84.6	22.1	87.4	24.3	91.9	22.1
	Long-term Decline Up 0.5% p.a.	85.7	22.9	89.2	25.8	94.3	22.9
	Long-term Decline Down 0.5% p.a.	85.2	22.5	87.3	24.2	90.4	22.4

On the central projection basis the life expectancies of a female born in 2006 is 92.5 years, while it is 91 years for a male. Again, a female aged 65 years in 2006 is projected to have a future life expectancy of 22.7 years, while it is 20.6 years for a male on the central projection basis.

The rule-of-thumb from the above table is that a change in the current rate of decline by 1%, will change projected period life expectancies by 1 year at age 0 and 0.7 years at age 65 (i.e., by 1.2% and 3% respectively). The change in cohort life expectancy in 2006 is about the same for 65 year olds but less at age 0 at 0.8 of a year (0.9%). A change to the long term rate of decline has a bigger influence the longer the projected period and, in particular, cohort life expectancies at younger ages are particularly sensitive to this assumption.

If we assume that the long-term rate of improvement is 3% per annum rather than the central projection's assumed 1.5% per annum. Then the above table suggests this change will increase the cohort life expectancy of a male born in 2006 to be about 97 years (in fact, the actual answer is 95.9 years) and that of a female to 97.9 years (the actual answer is 96.7 years).

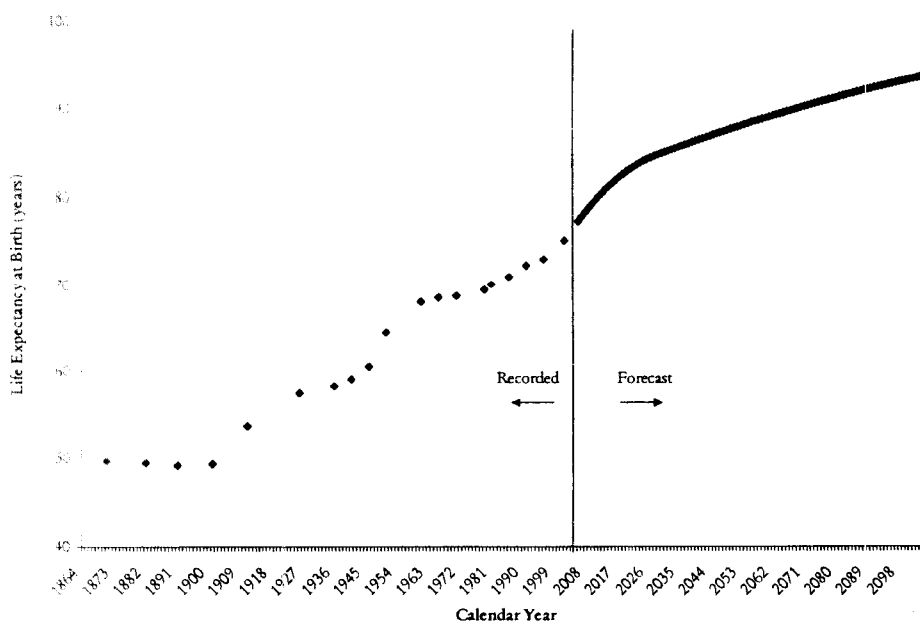
## Conclusion

We have analysed the patterns, both long-term and short-term, in the mortality experience of the Irish population. We identified a cohort effect, where those born in the 1930s have been experiencing a step-down in mortality rates as they age. The trend in mortality improvements has steepened significantly in more recent years. In particular, improvements are now being observed at the more advanced ages.

We briefly overviewed different methods of projecting mortality rates and applied two of the more popular of them to project Irish population mortality, each on several different bases. There is considerable uncertainty in the projections, as illustrated in *Tables 5 and 6*. However, it is not unreasonable to conclude, in answer to the question posed in the *Introduction*, that children born in 2006 can reasonably be expected to live to their nineties – the early nineties for males and mid-nineties for females.

Figure 9 sets out the recorded (period) life expectancy for a male borne in Ireland from 1871. The forecasted life expectancy graphed is based on the central projection basis of the targeting approach, as adopted in CSO (2008).

**Figure 9: Recorded and Forecast (Period) Life Expectancy at Birth for Male in Ireland**



We conclude by pointing out the growing need for better estimates of mortality rates at ages above 85 years, as current methods employed are not in accordance with best international practice.

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## ***Appendix 1: Data from which Irish Population Mortality is Estimated***

The continuous registration of deaths in Ireland<sup>3</sup> began in 1864, with each record of death including the sex, age, cause and location of death. To estimate mortality rates, one requires the number in the population corresponding to the number of deaths – in this case the population in Ireland sub-divided by sex, age and location. Censuses have been conducted in Ireland since 1821 but have been reasonably reliable only since 1841. Accordingly, the mortality experience in Ireland can be estimated from official sources from 1864.

Formal life tables, showing how mortality varies by age and sex have been prepared from the experience 1925-7 (Irish Life Table 1) and since that time a total of fourteen have been prepared, the most recent relating to the period 2001-3 (Irish Life Table 14). Summary statistics of life expectancies in the twenty-six countries of Ireland prior to independence are reported in the *Report of the Commission on Emigration and other Population Problems 1948-1954* (Table 79, p. 106).

In this paper, which is more concerned with projecting future mortality, we restrict our analysis to mortality trends since 1926. Five caveats must be made on the mortality experience as it is recorded in Irish Life Tables 1-14.

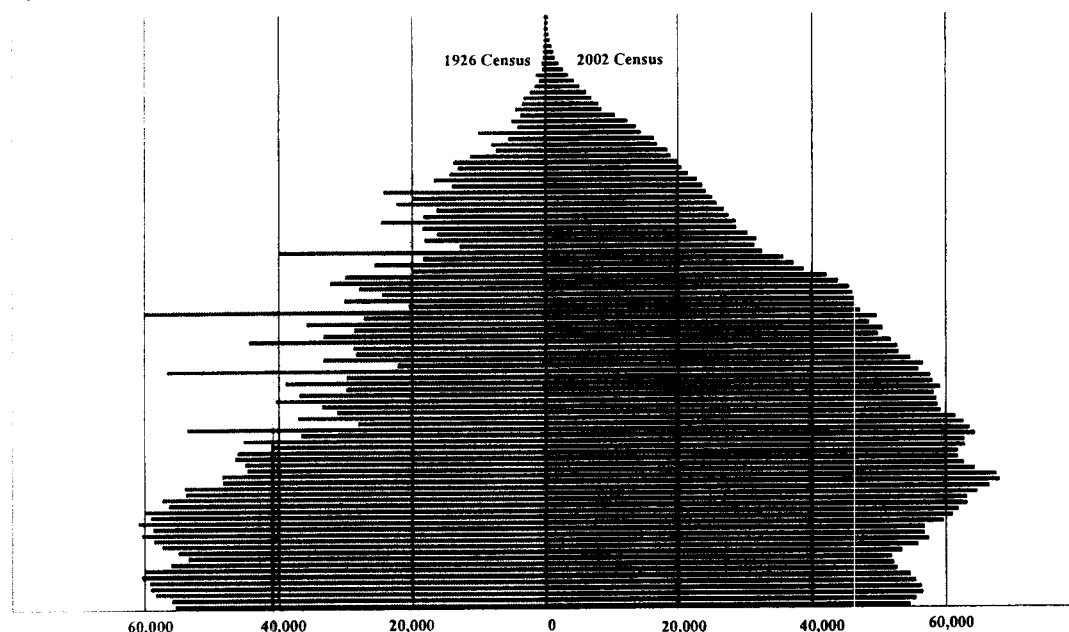
First, birth registrations up to 1941 are judged to be under-reported by 3-10% prior to 1941 but essentially complete after 1956 (Coward (1982)). This entails that infant mortality may be overstated somewhat prior to 1941.

Second, people when asked their ages at the regular censuses had a marked tendency in earlier times to round their ages to an age ending with 0 or 5, particularly at the older ages. This is a well-documented and internationally observed tendency known as ‘age heaping’. *Figure 1.1* shows the person count by age in the censuses of 1926 and 2002, with age heaping evident in the former and not the latter. The method of graduation of the Irish life tables has been designed to remove much of the affects of this rounding.

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<sup>3</sup> Acts for the official registration of births, deaths, and Roman Catholic marriages came into operation on 1<sup>st</sup> January 1864, from which time continuous records have been maintained and published annually in the *Annual Reports of the Register-General*. Registration of Protestant marriages began somewhat earlier, from 1<sup>st</sup> April 1845.

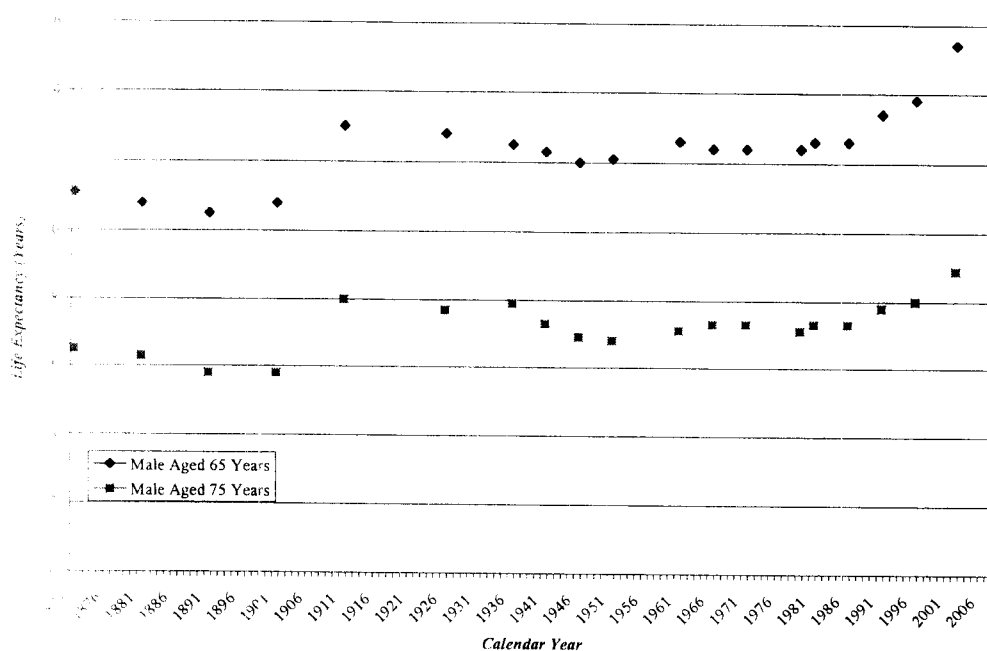
*Figure 1.1: Persons in Ireland by Reported Age in Censuses of 1926 and 2002.*



*Source: Based on data sourced from Central Statistics Office (CSO).*

Third, again in the earlier years, there may have been a tendency for older people to not just round their age but to exaggerate their age. Old age pensions were paid from 1909 to persons in the then United Kingdom over the age of 70 years, subject to means and other qualifying tests. While England registered births, deaths and marriages since 1836, Ireland as noted earlier, only began official records since 1864. Thus there was no formal means to verify ages of anyone over 45 years old in 1909 (Wood (1908)) and, as could be anticipated, successful claims for pensions in Ireland exceeded that budgeted and, in fact amounted to “117 per-cent of the number of seventy and over, less paupers; and this assumes that not a single person of seventy and over in Ireland has an income of £31 per annum” (see O.T. Falk’s discussion on Marr (1909), quote is from pp. 270-1) In fact, the expenditure over-run of pensions in Ireland was one of the main reasons for Lloyd George’s budget of 1909 that lead to the constitutional crisis (Ó Gráda (2002)). It could be expected that a person would report an age at subsequent censuses consistent with their declaration of age for pension. A male, say, aged 65 in 1909 claiming to be 70 years old could be 82 years old in 1926 and, according to Irish Life Table 1 Males, be alive in 1926 with probability 0.28. This individual would bias the estimated mortality rates downwards. We observe, consistent with hypothesis, that life expectancies for both Irish males and females at age 65 and 75 years show a suspicious jump between 1900-02 and 1910-12 of more than 20% and a decline thereafter. In fact, life expectancies for males aged 65 falls below the 13.0 years estimated in 1910-12 until 1990-2 and life expectancies at age 75 take until 1995-7 to regain the level estimated in 1910-12.

**Figure 1.2: Life Expectancies for Male Aged 65 and 75 Years, Ireland (26 Counties), Based on experience around the 3 years centred in calendar year shown**



Source: CSO (2004a), figures from Table 3.

The data for both deaths recorded and the exposed-to-risk population numbers in Ireland in respect of ages above 80 years, while improving with time, has been described even recently as “conditionally acceptable quality” with “data give probably a roughly correct description of the mortality trend though at a level artificially lowered by age overstatement” (Kannisto (1994)). The issue is that death certificates accurately report age at death but that age statements in census returns for the elderly tend to exaggerate the age – now for reasons other than the old age pension. Much of the problems can be overcome using the method of extinct generations (see, for instance, Humphrey (1970)), which bases the analysis on death records only, but such an investigation is beyond the scope of this paper.

The reliability of records in regard to the cause of death are also questionable: in the 1920s over one-quarter of deaths were not certified by a medical practitioner (Brown (1930, p. 101)) and even nowadays it is estimated that one-third of death certificates are likely to show incorrect cause of death (Roulson *et al.* (2005) quoted in O’Reilly (2006)). Only 26.5% of deaths in Ireland referred to coroner (O’Reilly (2006)).

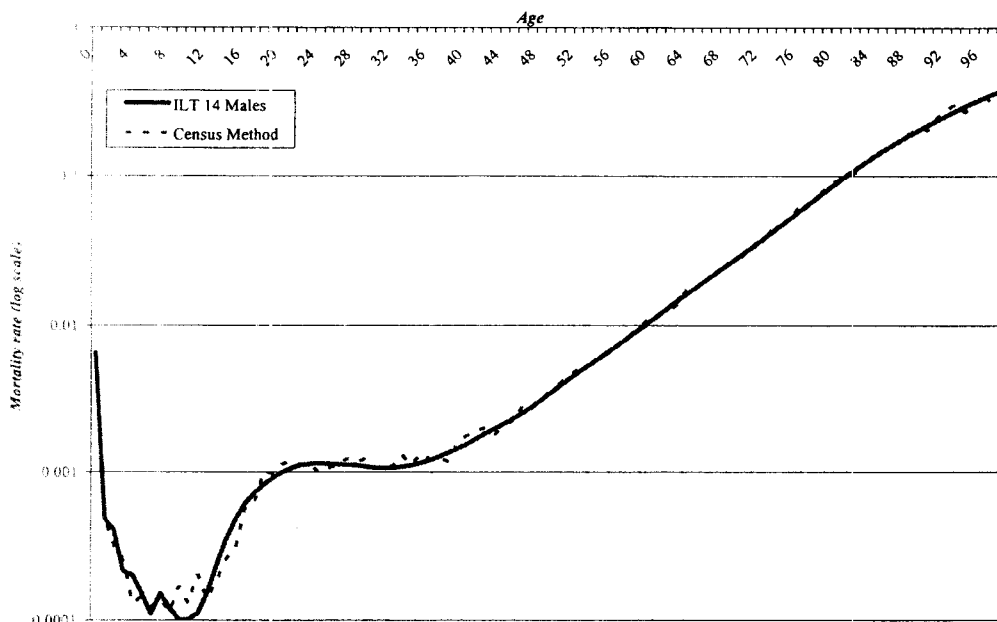
The method used to construct all 14 of the official Irish life is based on an old actuarial method, King’s Method (King (1909)), which was employed also to graduate life tables for England and Wales between 1901 and 1930-32 (ELT 7 to 10). The method involves

smoothing the series of deaths by age and population by age to reduce the effects of age heaping, before estimating mortality rates and is typically applied to data grouped in quinquennial age groups. Osculatory interpolation is then used to estimate intervening  $q_x$ s and at the extremes of age -- under 6 years and over 87 years - ad-hoc methods are employed. For instance, mortality rates above age 87 years are obtained by fitting a quadratic or Makeham curve (see Geary (1929), CSO (1986)).

Brown (1930) reviewed the construction of the first Irish life table. He judged that “Messrs Hooper and Geary in particular are to be congratulated on their enterprise to elucidated the obscurities of Irish population statistics” (p. 103) but that the data problems referred to above mean that the “Saorstat Life Table cannot be unreservedly accepted as a reliable index of actual conditions” (102-3) and, in particular, the exceptionally light mortality for both sexes at advanced ages might be partially because “as the pension age approaches the temptation to misstatement of age has still proved irresistible to a considerable section of the community” (p.102)

Investigation shows that King’s method as employed provides a reasonably smooth curve that closely fits the underlying crude specific age mortality rates. In fact, King’s method with osculatory interpolation can be viewed to be a fore-runner of modern spline graduation.

**Figure I.3: Irish Life Table 14 Males (2000-2002) graphed against Crude Mortality Rates estimated using Census Method.**



Sources: CSO (2004a) for graduated age specific mortality rates of males; crude mortality rates calculated by author based on the average deaths in three years 2000-03 and the number of males at that age enumerated in the Census of 2001 (using census method).

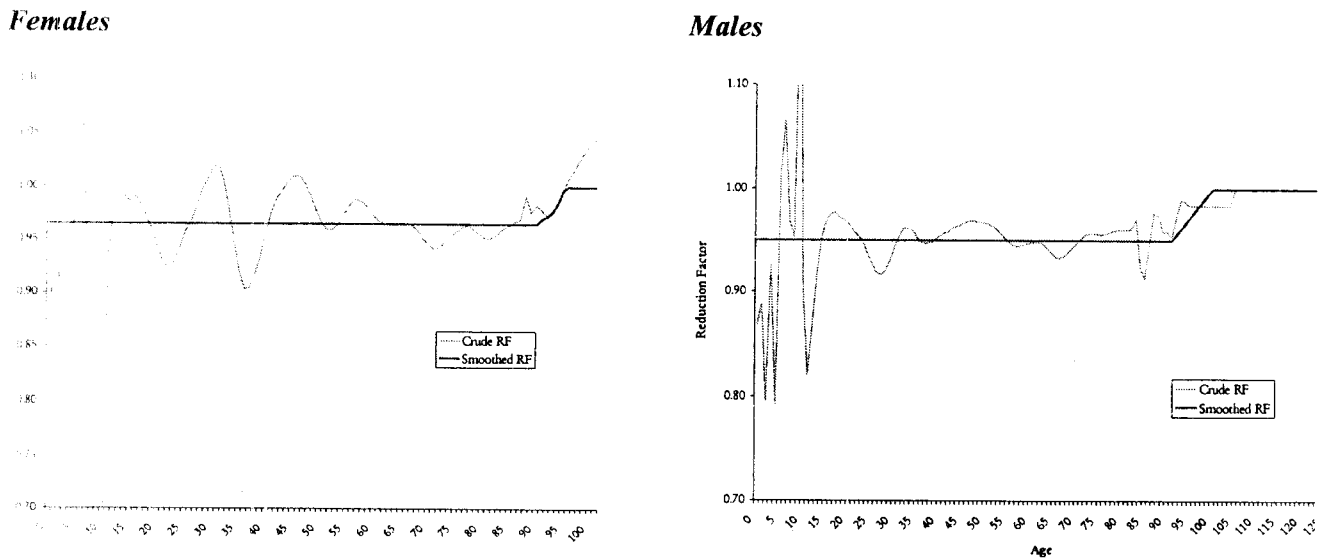
However, the ad-hoc graduation method applied at the older ages, and the census method approach to estimating crude rates is no longer satisfactory. It is now well established that the shape of mortality curve at advanced ages does not follow a Gompertz or Makeham curve but is better modelled using a logistic curve (Thatcher *et al.* (1998)). A better estimate is required for mortality at these advanced ages if only because more of the population can be expected to survive to these later ages.

We conclude that Irish life tables since 1926 give a best estimate of the mortality experienced based on the available data. Data quality had been improving over the years. There is an issue with estimating mortality at the highest ages due to age misstatements at censuses and, ideally, mortality rates above age 85 years or so could be better estimated, in line with international best practice, using the method of extinct generations and graduated using a logistic curve.

**Appendix II: Steps in forecasting Irish population mortality by target method**

1. Recent graduated life tables are prepared for each sex, separated by a short number of years. We used Irish Life Tables 14 (corresponding to morality experience 2001-2003, so centred in 2002) and, applying the same method, a graduated table was prepared for the experience 2004-2006, so centred in 2005.
2. The annualized percentage fall in mortality at each age for each sex was calculated from the graduated rates. This gave the average rate of improvement per annum over the three year period 2002 to 2005. These rates are then expressed as a reduction factor (RF), i.e., unity less the annual percentage rate of decline.
3. Figures 2 and 3 below graph these crude reduction factors by age. We notice that there is large fluctuations at the early ages but from age 11 to 90 or so the RFs tend to oscillate about 0.95 for males and, with somewhat greater amplitude, about 0.965 for females. In fact the mean of the crude RF from age 0 to 90 is 0.949 for males and 0.957 for females and from age 11 to 90 the means are 0.950 and 0.967 respectively.

**Figure II.1: Reduction Factors for Females & Males, Crude and Smoothed**



4. Most – almost all – projection methods in common use do not attempt to smooth the crude RF. This can lead to inconsistencies in the projections, with, say, age  $x+1$  having lower mortality than age  $x$  at some point in the future or, in general, the projections producing a very oddly shaped mortality curve. To avoid this we smoothed the RF factors across ages. In fact, we adopted a very strong smoothing approaching by

essentially replacing the RFs up to age 90 with their average rate. The smoothed RF was 0.95 for males and 0.965 for females, for all ages up to age 90 years. The ultimate justification for the strong smoothing adopted is that life expectancies at ages 0 up to age 65 years showed no significant differences in each future year whether the unsmoothed RF factors or the strongly smoothed factors were used. From age 100, we assumed the RF was 1 (i.e., no reduction) for both sexes.

5. The smoothed RF factor was assumed to apply to mortality rates between calendar year 2004 and 2005.
6. In 25 years' from 2005, (that is, from calendar year 2031) the reduction factor in any one year is assumed to be 0.985 for both males and females. This is justified on the basis that it is close to the average mortality improvement over the long term past and, as such, might be reasonably assumed in the long term future. The average annual rate of improvement over the 76 years, 1926 to 2002, was 1.4% for males (estimated as a simple average of rates of improvement at each age from 0 to 100 over that period). For females, the correspondingly average annual rate was higher at 2.1%. A simple average over the period would suggest a long term rate of decline were about 1.75% p.a. The selection of a rate is difficult and eventually a rate of 1.5% p.a. was settled on as it is close to long term past, and, when the other elements of the approach are included, produces estimates of period life expectancies in 2041 not very dissimilar to the last official projection.
7. In summary, the projection methodology is to assume that in any calendar year after 2031, the reduction in mortality over that year is 1.5% (i.e. apply reduction factor of 0.985 to the previous year's rates). In 2004 to 2005 the smoothed RF is assumed to apply, so 0.95 for males and 0.965 for females. For all years between 2005 and 2031, the RF for that year is a simple linear interpolation between these two extremes. The base table that the resultant cumulative RFs are applied to is the graduated life table for each sex, centred at calendar year 2004.